APPLIED PHYSICS LETTERS **86**, 254103 (2005)

Spectral response of a gigahertz-range nanomechanical oscillator

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(Received 28 September 2004; accepted 20 May 2005; published online 17 June 2005)

We report the measurement and simulation of the transverse displacement spectrum of a multi-element nanomechanical oscillator at previously inaccessible frequencies of up to 3 GHz. The detected displacement signal is enhanced in the high-frequency range by the presence of high-order resonance modes generated by coherent motion of individual elements. The spectrum reveals a rich structure with groups of peaks forming quasibands. The spectral structure is qualitatively analogous to atomic emission spectra. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1952847]

Nanomechanical oscillators with gigahertz-range natural frequencies are vital for testing quantum mechanics in the macroscopic limit.[1,2] Manufactured by standard semiconductor fabrication techniques, these high speed oscillators offer the ingenious possibility of being used as frequency-selective oscillators[3] and passive filters in microwave communications circuits,[4] memory elements,[5] and spin-mechanical devices in spintronics.[6] Essential to these applications of fundamental and technical interest is the control over the design and detection of structures with gigahertz-range frequencies.

Although the natural frequency of a simple doubly clamped beam can be increased arbitrarily by reducing the length, detection of the resonant motion quickly becomes problematic at gigahertz frequencies as the beam displacements become reduced to the femtometer scale due to the rapidly growing effective stiffness $k_{\text{eff}} \propto \omega^2$. Much of the recent experimental efforts have been devoted to measuring a single nanomechanical oscillator using ultrasensitive displacement detection. However, the current state-of-the-art displacement detection techniques[7,8] have sensitivities on the order of 10^{-15} m/Hz$^{-1/2}$, two orders of magnitude from the sensitivity required to detect quantum mechanical displacements in simple beam structures at gigahertz frequencies.

In a fundamentally different approach, we consider multi-element oscillators with multiple degrees of freedom to generate a richer spectrum of modes. The introduction of even a few periodic elements or internal degrees of freedom into a simple structure like a doubly clamped beam results in a modification of the frequency response to include series of high-order resonance modes. The effect is an enhancement of the displacement amplitude in the previously inaccessible gigahertz frequency range. Furthermore, the grouping of the high-order modes into quasibands and bands (in the limit of large numbers of coupled oscillators) is expected from the theory of linear oscillator chains. The frequency response spectrum then effectively maps out the energy bands at the equivalent Brillouin zone boundary.

Here we present the simulated and measured response spectra of a hybrid nanomechanical structure composed of arrays of coupled submicron cantilevers.[9] Finite-element simulation of the transverse displacement response at the midpoint of the structure reveals individual as well as quasibands of peaks separated by well-defined gaps, as shown in Fig. 1. Above 477 MHz, the prominent peaks in each band correspond to high-order modes with a high degree of phase coherence in the motion of individual coupled oscillators. Qualitative comparison of the response spectrum of our oscillator with atomic spectra of simple atoms like hydrogen or helium immediately reveals striking similarities, and suggests that the periodic array structure can indeed be thought of as a nanomechanical "superatom" possessing the basic spectral components and characteristics of simple atomic systems.

The single-crystal silicon oscillator is a two-element nanomechanical structure. The two arrays of 20 identical 0.5 μm long cantilevers are elastically coupled to a central 10.7 μm long doubly clamped beam. We fabricate and measure a set of four such identical oscillators at a temperature $T=250$ mK in a dilution cryostat. Each oscillator is driven magnetomotively via Lorentz force with current $I(\omega)$ through the central beam electrode of length L in magnetic field B. The measured signal is an induced voltage V_{emf} proportional to the beam momentum. Using simple classical harmonic oscillator expression, the voltage V_{emf} near resonance is given by $V_{\text{emf}}/\eta\omega BLx = (1/\sqrt{(\omega_0^2-\omega^2)^2+(\omega_0^2/Q)^2})\mathrm{BLI}(\omega)/m$, where η is an integration factor over the particular mode shape of the central beam. As verified in Figs. 3(c) and 3(d) for a typical high-order 2.34 GHz resonance peak, the driving force can be varied using both the driving current I_{dr} and the field B, and the resonance responds linearly to the former (for small drives) and shows quadratic dependence on the latter, according to the classical expression just presented.

Linear finite element simulation of the structure reveals a multitude of complex vibration modes, and we focus on the set of transverse out-of-plane mode shapes with odd numbers of antinodes, as these are expected to generate the largest response voltages in our measurement setup. In this set, all the prominent peaks below 477 MHz are overtones of the fundamental resonance mode at 6.4 MHz [see Fig. 1(b)], which generate a spectrum equivalent to a simple doubly clamped beam. We focus instead on the high-frequency set of resonance peaks that start at 477 MHz. As illustrated in Fig. 1(b), the mode shapes of certain high-order modes display phase-symmetric motion of the cantilever arrays, in

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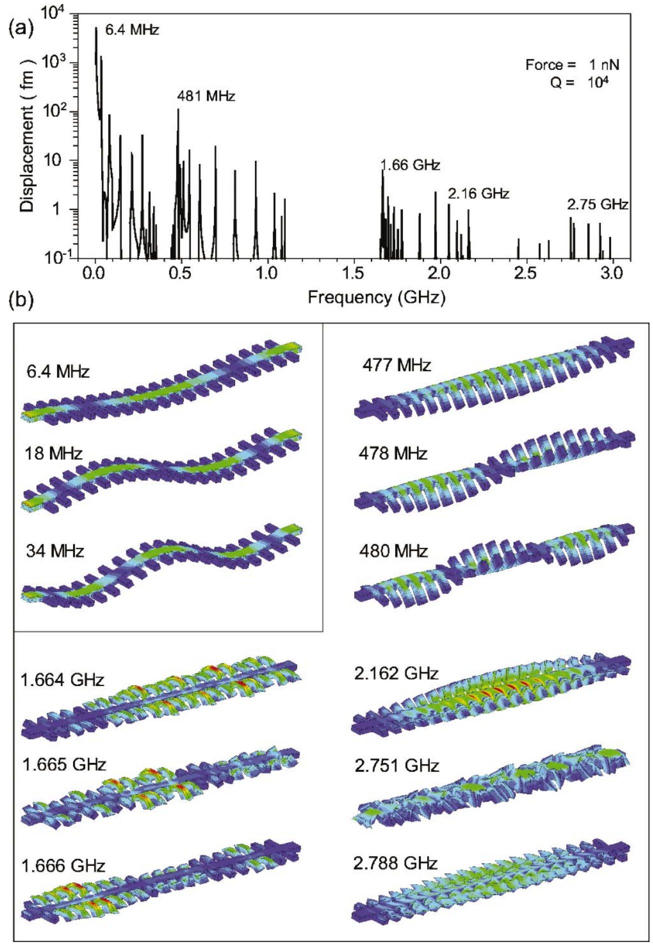

FIG. 1. (Color) Finite element simulation of the antenna structure. (a) The linear midpoint response spectrum of the oscillator is shown over the range 1 MHz to 3 GHz. The y axis corresponds to the out-of-plane displacement amplitude at the geometrical midpoint of the structure. Given the structure symmetry, the midpoint displacement at a single driving frequency is a single parameter that describes the net transverse area swept by the vibrating central beam—the quantity that is measured in our measurement setup. The spectrum of the periodic structure reveals several series of peaks, which correspond to resonance modes displaying a high degree of phase coherence among the cantilevers. For constant material damping $1/Q=10^{-4}$ and transverse harmonic driving force of $|F_{dr}|=1$ nN applied uniformly on the central beam, the displacements are enhanced from femtometers to the picometer range in some of the prominent high-frequency modes. (b) The associated mode shapes for the first few resonance peaks in each series are shown. Color coding indicates strain intensity in the linear elastic approximation. Note that the second and higher even modes in each series have negligible contribution to the calculated spectrum.

contrast to the majority of other complex modes where the cantilever motion is uncorrelated. These collective modes are analogous to Bloch oscillations of a set of two-level systems. Figure 1(a) shows the resulting frequency spectrum of these Bloch modes at frequencies above 477 MHz. The bands of response peaks correspond to the mode shapes with the greatest degree of phase symmetry among the cantilevers. The constant environmental damping factor of $1/Q=10^{-4}$ gives a width to each resonance peak and makes the peak height finite in comparison to an ideal oscillator with zero dissipation. The finite width of the response is similar to the relaxation rate of atomic emission spectral lines.

Figure 2(a) shows the response spectrum of our structure in the range of 1 MHz to 3 GHz, measured using an RF network analyzer. We identify several prominent response peaks that vary predictably with driving force. With Q values in the

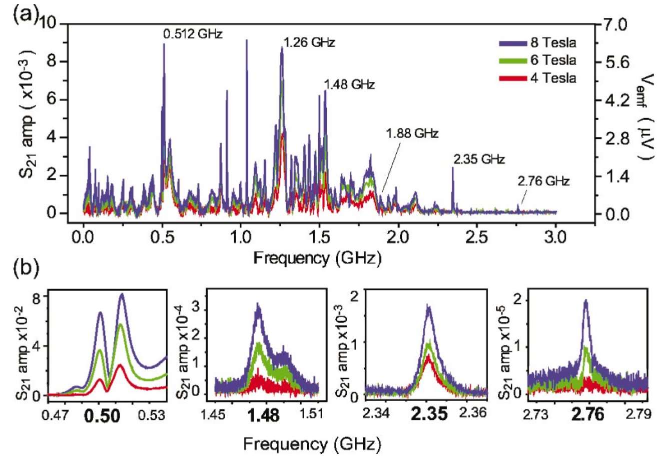

FIG. 2. (Color) The response spectrum of the antenna structure, measured at refrigerator temperature $T=250$ mK. (a) We plot the amplitude of the S_{21} transmission coefficient over a 3 GHz bandwidth for three different values of the driving force at three corresponding magnetic fields. S_{21} amplitude gives the detected voltage V_{emf}, which is proportional to the transverse displacements of the beam in the harmonic regime. (b) Prominent gigahertz-range resonance peaks show expected dependence on driving field. The location and relative response suggest that these peaks result from the first few Bloch modes in the simulated quasibands.

range of 500, the modes show significant mixing, which limits the frequency resolution of the measured spectrum [see Fig. 2(b)]. We characterize a typical high-order resonance at 2.34 GHz in Fig. 3, which shows the expected quadratic field dependence and linear Hooke's law force response. The highest detected mechanical resonance is shown in Fig. 2(b) at 2.76 GHz, improving our previous record[9] of 1.5 GHz.

Our approach of obtaining detectable high-frequency modes involves the introduction of periodic coupled oscilla-

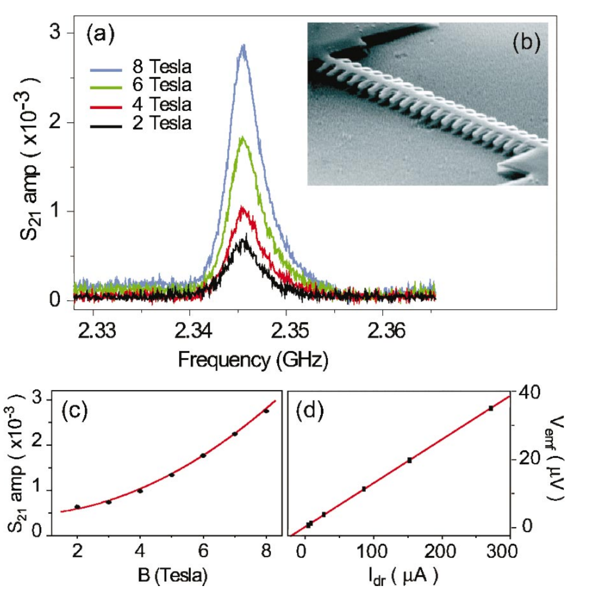

FIG. 3. (Color) High-order mechanical resonance mode of the antenna structure at 2.34 GHz. (a) We plot the Lorentzian line shape of the peak in the linear regime for various values of the driving field B. (b) The inset shows a scanning electron micrograph of the single-crystal silicon antenna structure, fabricated using standard e-beam lithography and nanomachining. The cantilevers are 200 nm wide with gaps of 300 nm between them. The central beam is 400 nm wide. The total uniform thickness is 245 nm, comprised of 185 nm silicon and 60 nm gold electrode. (b) At constant driving power, the peak height varies as B^2. (c) At constant field B, we verify the linear dependence of the measured signal V_{emf} on driving current I_{dr} for values below 1 mA.

tors rather than miniaturizing the simple beam structure. The coupled oscillator structure reveals a spectrum of well-separated resonance bands in the gigahertz frequency range. The rich spectrum is similar to atomic emission spectra, and it offers the possibility to explore mechanical motion at previously inaccessible microwave frequencies. Gigahertz nanomechanical oscillators hold the promise of entering into the quantum mechanical regime and could enable the long sought-after study of macroscopic quantum behavior, as well as quantum information processing with mechanical degrees of freedom.[10]

The authors acknowledge support by the NSF (DMR-0449670), and Petroleum Research Fund. Partial support of this work is also provided by the NSF (CCF-0432089 and DMR-0346707), the NSF NER program (ECS-0404206), the DOD/ARL (DAAD 19-00-2-0004), and the Sloan Foundation. The authors thank Nitin Shukla and Anil Saigal of Tufts University for collaboration on the simulation part of the project.

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