

Persistent current in normal metals

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Abstract. We discuss the recent experiments on persistent current in metallic rings in the backdrop of low temperature decoherence. The observed size of the persistent current, typically on the order of the Thouless energy, e/τ_D , is much larger than the theoretical results, obtained with or without electron interaction. In considering the phenomenology of both decoherence and persistent current, usually observed in similar systems, we argue towards a dynamic role played by decoherence in the generation of a persistent current. A field-induced phase shift from near-equilibrium high-frequency fluctuations— which otherwise gives rise to decoherence—under certain conditions of periodicity and asymmetry due to disorder, is argued to induce a steady state diffusion current on the order of e/τ_D , comparable to the observed persistent current.

Keywords: decoherence, persistent current, fluctuations

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1 Introduction

One of the most spectacular aspects of the quantum mechanics of an electron in a small conducting system is interference; This results in the modulation of conductance periodic in enclosed flux with a period of the fundamental flux quantum h/e —the effect dubbed as the Aharonov-Bohm effect in mesoscopic physics. Similarly, a small metallic ring threaded with an Aharonov-Bohm flux displays a thermodynamic persistent current, signifying quantum coherence of electrons in the ground state. But an unavoidable feature associated with any interference effect in mesoscopic system is decoherence. Inside a disordered conductor, the decoherence of an electron occurs due to its interaction with its environment: coupling to localized spins, electron-phonon interactions and electron-electron interactions, the latter dominating at low temperature. Though the coupling to an environment results in decoherence, it is quite feasible for the coupling to induce a nontrivial coherent effect, contingent upon other requirements.

The presence of a flux Φ modifies the boundary condition for the electron wave function in an isolated ring, requiring dependence of the *equilibrium* free energy F on Φ ; This results in an *equilibrium* current in the ring[1]:

$$I(\Phi) = -\delta F(\Phi)/\delta\Phi. \tag{1}$$

$I(\Phi)$ is periodic in flux with fundamental periodicity h/e , the flux quantum, and only exists in the presence of a magnetic field. For a disordered ring, the fundamental

harmonic (with flux periodicity h/e) is strongly suppressed, whereas the first harmonic (with flux periodicity $h/2e$) survives due to the contribution of time-reversed paths of the electron[2]. Another condition is that only those electrons whose wave functions are sufficiently extended to wrap around the ring carry the persistent current; the phase decoherence length is larger than the length of the ring,

$$L_\phi > L, \text{ or equivalently, } \tau_\phi > \tau_D, \quad (2)$$

where $\tau_\phi = L_\phi^2/D$, and the time for the diffusion of the electron around the ring, $\tau_D = L^2/D$; The current $I(\Phi)$ is not expected to decay once this condition is satisfied.

Persistent current in normal metals is already known to exist[3, 4, 5, 6], though verified only in a handful of experiments. Recent experimental results on the size both in the h/e and $h/2e$ components are about two orders of magnitude larger than the most hopeful theoretical result, obtained with the inclusion of electron interaction perturbatively[7]. The simultaneous requirement of explaining the observed diamagnetic sign of the $h/2e$ component makes the comparison with theory even worse. Without electron interaction, the size of the current is too small, with the wrong sign[8].

Keeping the inadequacy of numerous attempts in view[7, 8, 9, 10], and motivated by additional experimental observations, we explore a possible connection between the observed persistent current and decoherence[11, 12]. In an excursion from the fundamental premise of its existence as an *equilibrium* thermodynamic property, we argue that the observed current in the experiments may *not* be the textbook persistent current—an aspect of complete quantum coherence of electrons in the ground state *in equilibrium*. In stead it could perhaps be a steady state dc current generated out of coherent phase shifts due to the presence of near-equilibrium or non-equilibrium fluctuations. Our argument is motivated by the persistence of decoherence at low temperature[11] in the range where the persistent current is measured[3, 4, 6, 13]. Assumption of the low temperature decoherence implies the presence of time-dependent field fluctuations. These fluctuations induce random phase shifts in the electron wave function, resulting in decoherence upon averaging. In an isolated *periodic* structure, correlation can arise from these random field-induced phase shifts for frequencies of the field matching with the traversal frequency of the electron diffusion around the length of the periodic structure, i.e. for $\omega = 2\pi/\tau_D$. For these paths, the phase shift in the electron wave function induced by the field fluctuations is 2π ; The electrons can, in principle, carry energy from the field fluctuations, though a steady state finite current can only be maintained by the electrons in these paths due to the absence of decoherence. Other paths, or equivalently, frequencies higher or lower than $1/\tau_D$ contribute to a transient current which dies down within a time scale of $1/\tau_\phi$.

In the next section we report our experimental results on the first complete measurement of the persistent current, including comparisons with previous experiments. In the subsequent section we briefly review the previous theoretical attempts with and without electron interaction. Finally, we discuss the plausible dynamic role played by an environment of high-frequency electromagnetic field fluctuations. The environment is assumed to be arising from electron interaction perhaps in the strong coupling limit, though such an assumption is not necessary.

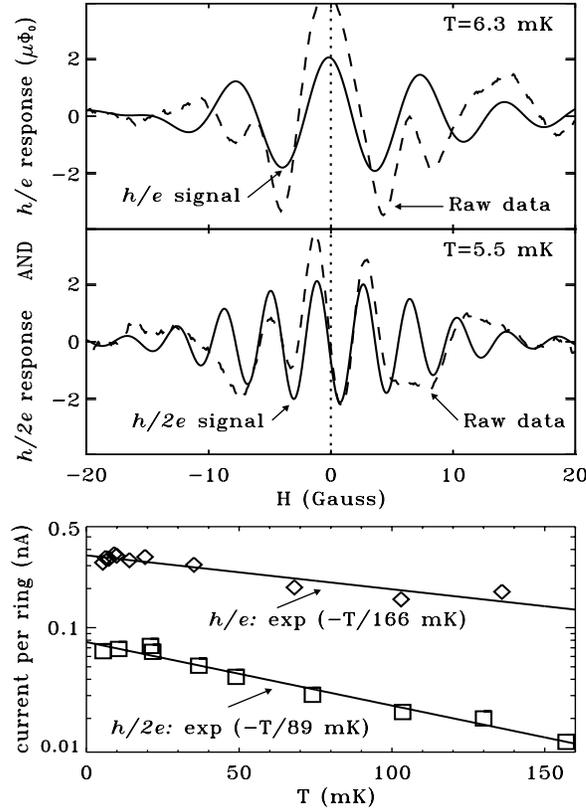


Fig. 1 The observed h/e - and $h/2e$ -flux periodic currents are displayed in the top panel. The bottom panel shows the temperature dependence of both components.

2 Phenomenology

Persistent current in normal metal systems has been observed only in a handful of experiments[3, 4, 6]. The magnitude of the h/e current along with its temperature dependence has been measured in experiments on single Au rings[4]. The h/e component of the current has also been observed in a semiconducting GaAs-AlGaAs heterostructure ring[5]. The $h/2e$ current along with its temperature dependence has been measured, in the first persistent current experiment[3], in an array of 10 million Cu rings. The h/e component measured in the ballistic ring[5] has the expected magnitude of ev_F/L , where v_F is the Fermi velocity. The size of the h/e current measured in single Au rings[4] is two orders of magnitude larger than the anticipated value of $ev_F/L(l_e/L)$ for the typical current in a single disordered ring; $l_e \sim D/v_F$ is the elastic mean free path denoting disorder. Likewise, the $h/2e$ current measured in the multiple Cu ring

experiment was more than two orders of magnitude larger in size than the anticipated values of $\sim e\Delta/\hbar$ for the $h/2e$ current averaged over disorder and many rings. The sign of the h/e current was determined to be random as expected, whereas the sign of the $h/2e$ was not determined. Temperature dependence of both the components of current was found to be exponentially decaying with increasing temperature.

In what follows, we describe the results of our recent experiment without going through the experimental details, published elsewhere[6]. We have measured the magnitude, temperature dependence, and sign near zero field of both the h/e - and $h/2e$ -periodic components of the persistent current in an array of 30 Au rings. Let us first summarize the results.

2.1 The $h/2e$ current

(a) The magnitude of $\sim 0.5e/\tau_D$ observed in the $h/2e$ channel is in agreement with the earlier experiment on Cu rings. (b) Temperature dependence of $e^{-T/89\text{mK}}$, though within a (limited) range of 10-150 mK, is comparable to previous measurements. (c) The sign of the current near zero field was determined to be diamagnetic. (d) We have observed a strong magnetic field dependence on all our $h/2e$ current traces with a characteristic field $H_c = \sqrt{3}\hbar/ewL$, where w is the width of the ring. This field dependence is perhaps due to the magnetic field penetration into the arms of the ring corresponding to a decay rate $1/\tau_H = D(eHw/\sqrt{3}\hbar)^2$, comparable to magnetic-field-induced phase-breaking in weak localization.

2.2 The h/e current

(a) We observe a magnitude of $2.3eE_c/\hbar \simeq 10^{-2}(ev_F/L)$ for the average h/e current. Unfortunately, a comparison with earlier experiments is not possible, since the average h/e current—in an ensemble of many rings—has not been measured previously. (b) The temperature dependence is found to be $e^{-T/166\text{mK}}$. (c) The sign of the current near zero field is diamagnetic. (d) The h/e current also showed a suppression of the size with increasing Φ , though the effect was weaker with a larger H_c compared to the H_c of the $h/2e$ current.

2.3 Sample parameters

The Au rings are described by the following parameters: $2.56 \pm 0.05\mu\text{m}$ in diameter, $120 \pm 20\text{nm}$ wide, and $60 \pm 2\text{nm}$ thick. Their resistance per square of $0.15\Omega/\text{square}$ corresponds to a diffusion constant of $D = v_F l_e/2 = 0.06\text{m}^2/\text{s}$, with an elastic mean free path of $l_e \simeq 87\text{nm}$. These parameters as well as L_ϕ of $16\mu\text{m}$ are obtained in a separate transport measurement of weak localization. The control sample used in this measurement is a $207\mu\text{m}$ long meander with the same thickness and linewidth as the rings, fabricated simultaneously. L_ϕ , or equivalently τ_ϕ , was found to be essentially temperature independent below 500 mK. Various energy scales of the disordered rings are the following: Thouless energy $E_c \sim \hbar/\tau_D = 7.3\text{mK}$, mean energy level spacing $\Delta \sim 1/2\mathcal{N}_0\mathcal{V} = 19\mu\text{K}$, where \mathcal{N}_0 is the density of states, and \mathcal{V} is the volume of each ring.

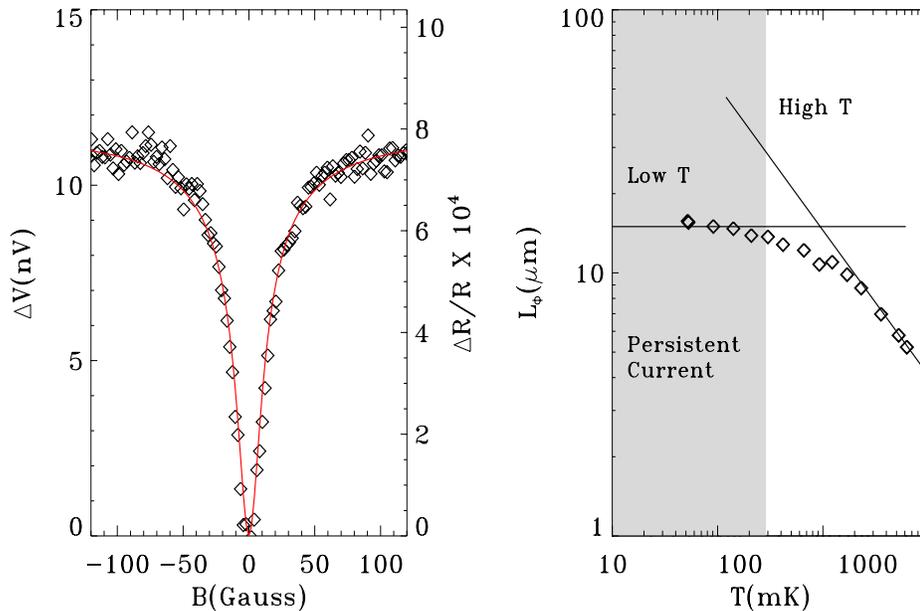


Fig. 2 (a) Weak localization measurement for the determination of L_ϕ . (b) Temperature dependence of L_ϕ in the corresponding control sample of quasi-1D Au wire. The shaded area represents the range of temperature in which the persistent current was measured.

2.4 Comparison with previous theories

(i) The observed magnitude of the $h/2e$ current of $\sim 0.5eE_c/\hbar$, along with the diamagnetic sign is hard to understand in previous theories[7, 8]; This result is much larger than the expected average current of $e\Delta/\hbar$ in non-interacting calculations[8], though a typical current on the order of eE_c/\hbar is expected. The first theoretical calculation that includes the electron-electron interaction[7] obtains a current within an order of magnitude of eE_c/\hbar , but it requires a repulsive Coulomb interaction, necessarily resulting in a paramagnetic current. In order to obtain a diamagnetic current in such a theory, a phonon-mediated attractive electron-electron interaction must be used; however, the small strength of the potential of such an interaction gives a result even smaller by another order of magnitude. (ii) We observe a magnitude of $2.3 eE_c/\hbar \simeq 10^{-2}(ev_F/L)$ for the h/e current in our ensemble of rings; in contrast, both non-interacting and interacting theories find that the average h/e current should be vanishingly small in the diffusive limit. To avoid any confusion, we note here that our measurement of the *average* h/e -periodic response from thirty rings is smaller than the magnitude ev_f/L of the *typical* h/e current measured in single rings[4]. For the typical current in a single ring, theory suggests an amplitude eE_c/\hbar , again much smaller than measured [4]. In either single[4] or multiple ring experiments[6], the measured amplitudes of the h/e response are much larger than accounted for by any theory. (iii) The exponential dependence for both h/e and $h/2e$ components on temperature is similar to what was found in earlier experiments, in both single [4] and multiple rings[3]. Even though some

theories[7, 8] contain a temperature dependence, one must be cautious in comparing our result to such predictions over a limited range of very low temperatures. (iv) Another interesting feature of our experiment, common with previous experiments, that decoherence rate $1/\tau_\phi$ is completely saturated[13, 4, 6], in the range of 10-150 mK in which the persistent current is measured in the Au rings.

3 Magnitude and sign of the current in previous theories

3.1 Single electron picture

In order to understand all the experimentally observed features of the persistent current consistently, it is important to briefly review the earlier attempts. The simplest model for persistent current from an isolated system of electrons in a disordered potential involves the modification by the flux Φ of the single-electron energy spectrum—neglecting the interaction between the electrons[8, 9]. Naively, though incorrectly, it can be assumed that the thermodynamics of the electron system is governed only by the mean spacing Δ of energy levels of the electron eigenstates. The flux dependence of the total energy $E(\Phi)$ can be assessed as the sum of all occupied single-particle levels. Since $\partial E_i(\Phi)/\partial\Phi$ alternates in sign for consecutive levels, cancellation in the sum leads to a current defined by the last term around the Fermi energy E_F :

$$I(\Phi) = \sum_i \frac{\partial E_i(\Phi)}{\partial\Phi} \sim \frac{\Delta}{h/e}. \quad (3)$$

This is a strange result, since the level spacing is not affected by disorder(D). Interference corrections such as weak localization modify D , leaving Δ unaffected. Thus in the single electron picture thermodynamics of the ring is disorder independent and is not due to quantum interference. It can be argued that thermodynamic properties of small systems at low temperature are governed by the mesoscopic fluctuations rather than the equilibrium distribution of the single-electron energy levels near E_F . But fluctuations in the distribution give negligible contribution to any thermodynamic quantity in an ensemble where the chemical potential μ is fixed. In a canonical ensemble with the number of electrons N fixed, however, non-negligible corrections to the mean value of thermodynamic quantities such as the persistent current. A trick is used, to allow for the calculation of the current by conventional methods;

$$\langle\langle \frac{\partial F(N, \Phi)}{\partial\Phi} \rangle\rangle_N \equiv \frac{\Delta}{2} \frac{\partial}{\partial\Phi} \langle(\delta N)^2\rangle_{\mu=\langle\mu\rangle}, \quad (4)$$

which signifies an elegant identity that the current as the flux derivative of the free energy in a canonical ensemble is equivalent to the flux derivative of the variance in the number fluctuation $\langle(\delta N)^2\rangle$ in the corresponding grand canonical ensemble. The number fluctuation in a disordered conductor has been studied in great detail, and has been found in the presence of a flux Φ —which breaks the time-reversal symmetry—to be

$$\langle(\delta N(E))^2\rangle = \frac{4}{\pi^2} \left[\ln \frac{E}{\Delta} + \ln \frac{E\Phi_0^2}{E_c\Phi^2} \right], \quad (5)$$

valid for $\delta < E_c(\Phi/\Phi_0)^2 < E_c$. using this one obtains an expression for various harmonics of the current $\langle I(\Phi) \rangle = \sum_m I_m e^{2im\Phi/\Phi_0}$, periodic in flux:

$$I_m = \frac{4i\Delta}{\pi\Phi_0} e^{-2m/\sqrt{E_c\tau_\phi}} \text{sgn}(m). \quad (6)$$

The current, obtained in this calculation, is weakly dependent on disorder, through an exponential factor; the same factor introduces the temperature dependence as well by the replacement of τ_ϕ by $\hbar/k_B T$ at finite T . The maximum value of the current $4\Delta/\pi\Phi_0 \sim e\Delta/\hbar$ remarkably depends, once again, on the mean energy level spacing Δ of the single-electron eigen states. The current is observed to be paramagnetic near zero flux in this model. Experimental observation of a current of $eE_c/\hbar = e/\tau_D$, more than two orders of magnitude larger than $e\Delta/\hbar$, and the observed sign are contrary to the expectation. In the presence of strong spin-orbit scattering, as in Au rings, the expression for the $\langle (\delta N(E))^2 \rangle$ is slightly modified, and the sign of the current remains unchanged, failing on both counts to explain the experimental results.

3.2 Inclusion of electron interaction

In view of the experimental results it is imperative to consider interaction among the electrons[7]. Coulomb interaction among electrons give an orbital magnetic response[2] which is paramagnetic usually for a normal metal with repulsive interaction and diamagnetic for an attractive effective interaction—analogue to superconducting fluctuations in the normal state at a temperature slightly above T_c . In the first order perturbation theory, the interaction-induced persistent current[7] is obtained to be

$$I_m \sim (e/\hbar)\lambda_c(T)E_c, \quad (7)$$

which effectively replaces the mean level spacing, $\Delta \rightarrow \lambda_c(T)E_c$. The coupling constant $\lambda_c(T)$ is related to the bare coupling constant: $\lambda_c(T)^{-1} = \lambda_0^{-1} + \ln(\epsilon_c/k_B T)$, and the cut-off energy ϵ_c is typically E_F or the Debye temperature. Using $\lambda_c \simeq 0.08$ for Au, one obtains a current of an order of magnitude smaller than the experimental value. The perturbative treatment of the Coulomb interaction thus fails to explain the experimental results. In the absence of a nonperturbative analysis of the interaction-induced current, a different view towards the problem is required.

4 Dynamic role of the electron-field coupling in persistent current

Phase coherence is an important consideration in the existence of persistent current. Even inside an isolated loop, an electron possesses a finite decoherence time τ_ϕ , often slightly longer than the time taken for the electron to diffuse around the ring, τ_D . A question arises as to how these short timescales τ_ϕ and τ_D reconcile with the timescale of persistence, almost infinite. It is normally assumed, *ad hoc*, that the current, the flux derivative of the appropriate energy $I(\Phi) = -\partial F/\partial\Phi$, in a phase coherent ring with $L > L_\phi$ reflects absolute quantum coherence of each state. This conceptual assumption in some sense implies the absence of decoherence, or $1/\tau_\phi \rightarrow 0$.

Let us consider a specific decohering environment and analyze its role towards the generation of a current[16]. Though the following analysis is valid for any high-frequency environment[17, 18], we are interested specifically in the electromagnetic field[14, 15] created by the presence of other moving electrons, described by fluctuations with time-dependent correlation function. If the number of relevant degrees of freedom describing this field is small, thermodynamically speaking, or if the coupling with the electron is strong, then this decohering environment cannot be described as a true thermodynamic bath. This is because of competing timescales: Switching on the coupling between the electron and the environment disturbs the equilibrium of the latter. The fluctuations, described by time-dependent correlations, die down on a relaxation timescale which may not necessarily be short compared to τ_ϕ or τ_D , in which case the electron experiences truly non- or near-equilibrium fluctuations.

The time-dependent field induces a phase shift in the electron's wave function. In order to study the phase shift due to an electromagnetic potential in the case of a pair of time-reversed paths ($h/2e$) or any two interfering paths (or h/e) during a period of interaction in a temporal interval of $[t_i, t_f]$, the time dependence is defined in terms of sum and difference coordinates[19], $\bar{t} = (t_f + t_i)/2$; $t_0 = t_f - t_i$, and thus $t = \bar{t} + t'$; and $-t_0/2 \leq t' \leq t_0/2$. For an equivalent pair of time-reversed paths, $r_{t'}^{cl}$ and $r_{-t'}^{cl}$, the phase difference due to a time-dependent potential is given by

$$\delta\phi[r_{t'}] = -\frac{ie}{\hbar} \int_{-t_0/2}^{t_0/2} dt' [\Phi(r_{t'}; \bar{t} - t') - \Phi(r_{t'}; \bar{t} + t')], \quad (8)$$

where the time-dependent electromagnetic potential $\Phi = \int_T E(t)\dot{r}(t)dt$. One obtains for the phase shift:

$$\delta\phi[r_{t'}] = -\frac{ie}{\hbar} \int_{-t_0/2}^{t_0/2} dt \int_{-t_0/2}^t dt' E(r_{t'}; \bar{t} - t') (|\dot{r}_1(t')| - |\dot{r}_2(t')|). \quad (9)$$

An associated current j can be defined via a current density $j = \rho(|\dot{r}_1|e^{i\phi_1} - |\dot{r}_2|e^{i\phi_2})$, written as $[(e^{-i\phi_1} + e^{-i\phi_2})(|\dot{r}_1|e^{i\phi_1} - |\dot{r}_2|e^{i\phi_2}) + h.c.]$ or equivalently,

$$j \propto (|\dot{r}_1| - |\dot{r}_2|)[\text{constant} + \cos(\phi_1 - \phi_2)]. \quad (10)$$

The phase shift in Eq.9 is also proportional to the velocity difference between the two paths; the current depends quadratically on the velocities, and thus survives ensemble averaging:

$$\langle j \rangle = -\sin(4\pi\Phi/\phi_0)\langle \delta\phi(|\dot{r}_1| - |\dot{r}_2|) \rangle, \quad (11)$$

whereas the phase shift, being a random quantity, is zero upon ensemble averaging. This simple but elegant form has been derived and analyzed in detail earlier[16]. Naively speaking, the current depends on the average of the mean square velocity difference. The external field, in presence of a flux, generates a drift current in both directions. The symmetry breaking, required for a finite mean square average, is provided by the lack of inversion symmetry in the ring due to disorder. This asymmetry of the disorder potential is very crucial to the generation of the current. Such non-equilibrium currents in transport configuration have been shown to exist as the

so-called photo-voltaic effect[20]. The degree of asymmetry is a random function of the electron energy and it varies on a scale of \hbar/τ_D . Thus contributions from different correlated energy intervals of width \hbar/τ_D will fluctuate in sign and cancel each other, suggesting that the total current be on the order of $(e/\hbar)(\hbar/\tau_D) = e/\tau_D$. Note the similarity to the single electron picture[8], but with a different energy width interval for cancellation.

Effect of the high-frequency field on the electron causes random phase shifts which on averaging give a finite decoherence rate. But for a certain frequency $\omega = 2\pi/\tau_D$, the phase of the electron coincides with its initial phase after a diffusion time τ_D around the ring coincides. In such a case, randomization of the phase by averaging does not apply, and for this mode the current survives without dissipating. This is because it corresponds to a phase shift of 2π during each trip around the ring. This also suggests a maximum current at $\omega = 2\pi/\tau_D$ for a single electron diffusing in a time τ_D ; the size of the current roughly equals e/τ_D . More detailed calculations[16] shows that the maximum current obtained is $\sim 0.53(e/\tau_D)$, in truly excellent agreement with the measured value of $0.5(e/\tau_D)$. This dc response, arising as a rectification of high frequency fluctuations, is a steady state current in contrast to a thermodynamic current. Conceptually, a steady state current implies that there is no net energy transfer between the bath and the electron in this state, whereas an equilibrium state obeys the condition of detailed balance and is described by a Boltzmann distribution. Furthermore, the current, for a pair of time-reversed paths, is diamagnetic, also in agreement with our experiment. Though a slow temperature dependence is anticipated in theory, and the same is observed in experiments, the limited range of data must be taken into account for agreement.

5 Conclusion

We discussed the results of the experiments on persistent current in normal metal systems. We find that both the size and the sign of the observed current can be explained as a steady state dc response of a single electron to a high-frequency electric field. This field, assumed to be intrinsic to the metallic system, is perhaps generated by the motion of other electrons in the system. The time dependence of fluctuations in the near-equilibrium state, argued to be important in the strong coupling case, gives rise to this steady state dc current. Furthermore, this steady state current is microscopically argued to be in a non-dephasing resonant mode. The agreement between this theory describing a persistent dc response and the observed current in experiments is truly excellent.

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