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# Measurement of small forces in micron-sized resonators

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**Abstract**

We report the measurement of average response of micromechanical resonators to very small driving forces, with a corresponding energy resolution of the order of $\hbar\omega_0$, the energy quantum of the classical resonator system. The measured coherent excitation is more than an order of magnitude weaker than the random thermal fluctuations. 

*Keywords:* Fundamental forces; Extra dimensions; Suspended structures; Quantum limit

Measurement of forces of very small magnitude is important for reasons of both fundamental [1,2] and technical [3–6] interest. Exploration of small forces such as those hypothetically arising from the effect of *extra dimensions* requires measurements, for instance, of gravitational force on the submillimeter scale [1,2]. However, what is of immediate interest, in micron-sized suspended structures, is the prospect of observing macroscopic quantum effects [7], mainly due to the small size of resonators. To that end, we report measurements of small forces in micron-sized suspended GaAs structures. In our measurement the minimum detectable force is $48\times10^{-18}$ N at a resonance frequency of 1 MHz. The relevant bandwidth of 80 Hz in our measurement is determined by the intrinsic dissipation in the resonator. Our result is an overall improvement over other techniques [8,9] used in gravitational wave detection.

We report measurements on a two-element torsional resonator fabricated in a GaAs–AlAs–GaAs heterostructure by surface micromachining. We were able to measure four normal modes, as expected from a finite element simulation of the structure. The results described here correspond to a torsional mode at 1.0156 MHz in which two elements are twisting symmetrically about the long axis of the structure. Weak drive forces are generated by applying an AC current through a gold electrode on the surface, in the presence of an external magnetic field, while motion is detected by the voltage induced across the electrode [10,11]. The current is provided by an attenuated network analyzer output signal. The voltage across the electrode includes a resonance peak due to the motion and a baseline due to the electrode's resistance. The drive current is swept through the resonance peak while the voltage is fed through a preamplifier to the network analyzer input.

The important feature of this measurement is that the detection is coherent with respect to the excitation. Since the network analyzer records both amplitude and phase, it is possible to reduce the detector noise by averaging repeated measurements at a given phase of the response. Without this effectively lock-in technique our force and displacement detection would be limited either by the noise at the input stage of the preamplifier, or by the mechanical Johnson noise of the structure, depending on the temperature. In our structure thermal fluctuations on resonance at 10 K correspond to a resonator displacement of $6\times10^{-3}$ Å. At this temperature, the preamplifier is the dominant noise source, with an equivalent force noise of 2.4 $\mathrm{fN}/\sqrt{\mathrm{Hz}}$. This is larger than the force noise of $5.6\times10^{-18}$ $\mathrm{N}/\sqrt{\mathrm{Hz}}$ previously attained at a much lower frequency of 1.7 kHz [12]. To demonstrate the utility of the coherent excitation and detection scheme, we applied a calibrated r.m.s. test force of $210\times10^{-18}$ N, and the mechanical response to this force was measured by the generated voltage across the surficial electrode. An anticipated signal on the order of 0.1 nV at the resonance frequency of 1.0156 MHz was detectable after averaging

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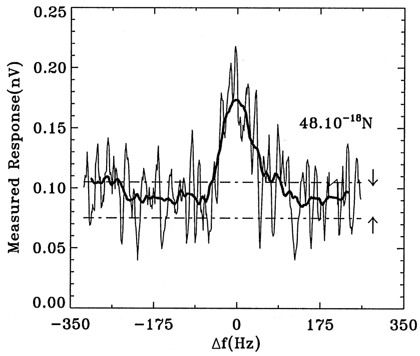


Fig. 1. The thin line is the response to an applied r.m.s. force of $210 \times 10^{-18}$ N, averaged over 2560 traces. The signal is detected at a central frequency of 1.0156 MHz corresponding to a mode of the structure. The noise level corresponds to a minimum detectable force of $48 \times 10^{-18}$ N with an intrinsic bandwidth of 80 Hz. The thick line is the smoothed trace.

2560 traces, with each trace having an average voltage noise of 1.6 nV. After averaging the noise floor, as shown in Fig. 1, was reduced to 23 pV. This corresponds to a total force noise of $48 \times 10^{-18}$ N. The signal on resonance corresponds to an applied r.m.s. force of $210 \times 10^{-18}$ N. The resonance, as shown in Fig. 1, has the expected intrinsic bandwidth $\omega_0/4Q = 80$ Hz for a measured quality factor of $Q = 20\,000$. It is important to note that due to the fast response time ($Q/\omega_0 \sim 3$ ms) of the resonator, averaging can done much more rapidly than in previous force measurements [6].

The minimum detectable coherent force of $48 \times 10^{-18}$ N corresponds to a displacement of $24 \times 10^{-5}$ Å, roughly 25 times smaller than the thermomechanical noise; this coherent force corresponds to a mechanical energy of $34\hbar\omega_0$. We intend to improve this energy resolution by increasing the resonance frequency $\omega_0$ and by reducing the resonator mass.

In conclusion, we have measured the response to a small force with a minimum detectable force of $48 \times 10^{-18}$ N using a coherent excitation and detection technique. This technique is applicable to the measurement of Casimir forces and gravitational forces where the distance between two plates or bodies can be changed coherently with the response detection. We believe such finite frequency measurement will be important in the observation of hypothetical forces arising in small length scales.

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