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Decoherence and quantum fluctuations

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We show that the zero-point fluctuations of the intrinsic electromagnetic environment limit the phase-coherence time in all mesoscopic systems at low temperatures. We derive this quantum-noise-limited dephasing time and its temperature dependence in the crossover to the thermal regime. Our results agree well with most experiments in one-dimensional systems. [S0163-1829(97)50120-2]

The importance of decoherence in the quantum to classical transition is well known.[1] There are many examples where a quantum system undergoes environmentally induced decoherence, such as the electron wave function in a low-dimensional solid, an atom trapped in a quantum optical system, and an isolated quantum system coupled to a measuring apparatus. It has also been suggested that the decoherence which must have occurred between the quantum and classical regimes during the inflationary era of the Universe was driven by quantum fluctuations.[1] Under certain conditions, quantum zero-point fluctuations of the electromagnetic environment can also cause decoherence, even at finite temperatures, and dominate over the thermal contributions.[2] These zero-point fluctuations, which persist even in vacuum are responsible for the Lamb shift, the natural linewidths of energy levels, the Casimir force, and the broadening of resonance lines in neutron scattering in solids.[3] In addition, it has recently been shown experimentally that these zero-point fluctuations might play an important role in the interference effects studied in mesoscopic systems.[4]

In quantum theory, the probabilistic amplitudes are added up using the linear superposition principle. The net probability contains along with the classical probability, the interference, or the off-diagonal terms of the corresponding density matrix which are characteristics of the quantum nature of the system.[5] The natural way to define decoherence quantitatively is to obtain a time scale τ_ϕ over which interference effects are suppressed, or the relative phases in different Feynman paths of the electron wave function are randomized. This is essentially the time over which an electron maintains its phase memory. Normally, phase randomization of the electron wave function in a mesoscopic system occurs due to inelastic scattering effects such as electron-electron, electron-phonon, and magnetic impurity interactions,[6–9] all of which can be considered environmental for the electron under study. We suggest, however, that zero-point fluctuations also contribute to decoherence of the electron wave function, resulting in physically observable effects, such as the suppression of interference phenomena in mesoscopic systems.

In this paper we show how zero-point fluctuations can cause dephasing in a mesoscopic system. For a one-dimensional (1D) quantum wire, we present two approaches for obtaining the zero-point-limited dephasing time τ_0 at low temperatures which are in good agreement with the measured saturation values of τ_ϕ found in many experiments. We find that τ_0 essentially depends on the diffusion constant D and the resistance per unit length of the sample under study. We also derive the functional form of $\tau_\phi(T)$ which is valid from the zero-temperature quantum regime into the high-temperature regime, and accurately describes the observed behavior in the quantum to classical crossover regime.

An example of the problem we are trying to understand is the temperature dependence of the phase coherence time τ_ϕ in a quasi-1D wire illustrated in Fig. 1. The gold wire has a resistance of 271 Ω, it is 207 μm long, 0.11 μm wide, 0.06 μm thick, and has a classical diffusion constant $D=0.068$ m^2/s. At the lowest temperature displayed, the phase coherence length L_ϕ determined from standard weak localization measurements[6] is 15.7 μm and the phase coherence time $\tau_\phi=L_\phi^2/D=3.6$ ns. Contrary to every theoretical prediction, τ_ϕ is essentially temperature independent at low temperatures, and this type of behavior is seen in every experiment on 1D wires and 2D films; however, the temperature at which the saturation behavior starts varies from 20 mK to 10 K depending on the system under investigation.[4,10–13] It has been shown that this saturation behavior is not due to magnetic impurity scattering, or heating of the electrons due to noise or excess power dissipation from the external environment.[4] The solid line drawn through the data was experimentally determined to be

$$\tau_\phi=\tau_0\tanh\left[\alpha\pi^2\left(\frac{\hbar}{\tau_0 k_B T}\right)^{1/2}\right], \tag{1}$$

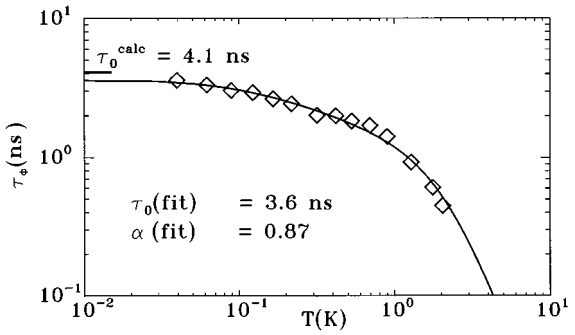

FIG. 1. Temperature dependence of τ_ϕ for a quasi-1D gold wire. The solid line is a fit to Eq. (1) with phonons. Our predicted τ_0^{calc} from Eq. (6) is shown on the plot.

where τ_0 is the low-temperature saturation value of τ_ϕ. Equation (1) was shown to fit most 1D experiments on mesoscopic metal wires with constant α only varying from 0.6 to 1.1.[4]

The functional form of Eq. (1) suggests that the zero-point fluctuations in the electromagnetic environment are responsible for the decoherence of τ_ϕ found at low temperatures in many experiments in mesoscopics. The zero-point field fluctuations can also be interpreted as a source field effect, which means that one can equivalently study a quantum system coupled to a fluctuating electromagnetic source field. The generalized spectral density function which includes the zero-point fluctuations of the electromagnetic environment[14,15] is

$$S(\omega)=\hbar\omega\left(\frac{1}{2}+\frac{1}{e^{\hbar\omega/k_BT}-1}\right). \tag{2}$$

At low temperatures, the zero-point fluctuations dominate, and $S(\omega)$ becomes temperature independent. The quantum system equilibrates with a quantum-statistical system described by the zero-point fluctuations. At high temperatures, $S(\omega)\rightarrow k_BT$, thus losing the quantum signature $\hbar$ of the electromagnetic fluctuations, and $S(\omega)$ is dominated by the thermal fluctuations. The quantum system in this case relaxes into equilibrium with a thermal bath at temperature T. In the intermediate regime, the full form for $S(\omega)$ must be retained in order to describe the transition between these two regimes.

We make a brief remark about what happens at low temperatures. When the system or a particle is coupled to an environment which possesses many degrees of freedom, even for weak couplings, the Hamiltonian for the entire system cannot be exactly diagonalized in the eigenbasis of the particle. The lack of a diagonal basis is the reason that the system can only evolve to mixed states even at zero temperature. One may construct, or in principle it is possible to realize in a physical system, an orthogonal basis for the full Hamiltonian in which the system may continue to stay in a well-defined stationary state. The choice of an initial state is also as important as the interaction. Usually an eigenstate of the full Hamiltonian is not easily prepared in realistic systems. Hence the possibility of a transient behavior of the system as it comes into contact with a temperature bath cannot be ruled out.

To estimate dephasing quantitatively, it suffices to evaluate the phase change of the electron wave function in a fluctuating electromagnetic field. It is well known that electron-electron interaction[6,7] at low temperatures may be studied by equivalently calculating the fluctuations in the electromagnetic field due to the other electrons which serve as the environment for the electron under study. The time dependence of this electromagnetic field causes decoherence. This is also equivalent to studying the changes induced in the states of the environment, as first pointed out by Chakravarty and Schmid.[7] The equivalence, which is merely an artifact of the fluctuation-dissipation theorem, was very nicely elucidated later by Stern, Aharonov, and Imry.[8]

We express the acquired phase ϕ in the electron wave function as the time integral of the interaction potential of the type $V[x(t),t]=\dot{\mathbf{x}}\cdot\mathbf{A}$ over the period of interaction. The quantity of interest is the average of the phase factor $\langle e^{i\phi}\rangle$. Here $\langle\ \rangle$ implies averaging due to disorder and field fluctuations which include both quantum and thermal parts. $\langle e^{i\phi}\rangle$ can be approximated as $e^{-(1/2)\langle\phi^2\rangle}$, because the mean value $\langle\exp(\alpha_i x_i)\rangle$ is given by $\exp(\frac{1}{2}\alpha_i\alpha_j\langle x_i x_j\rangle)$ where α_i are constants and the x_i are fluctuating quantities subject to a Gaussian distribution.[15] The suppression of the intensity of interference by dephasing is of the form $e^{-\langle\phi^2\rangle/2}$ which, for $\langle\phi^2\rangle\sim t$,[7,8] becomes e^{-t/τ_ϕ}. Generally $\langle\phi^2\rangle$ is not a linear function of time. One has to estimate the evolution of the off-diagonal terms of the reduced density matrix for the particle. The characteristic time over which $\langle\phi^2\rangle$ becomes of the order of unity can be defined as the corresponding dephasing time. We evaluate $\langle\phi^2\rangle$ by calculating the two time integrals of the function $\dot{\mathbf{x}}_\alpha\dot{\mathbf{x}}_\beta\langle A_\alpha A_\beta\rangle$.[6–8] Using the fluctuation-dissipation theorem in a gauge where the scalar potential is chosen to be zero one obtains the two-point correlation function for the vector potential $\langle A_\alpha(x(t),t)A_\beta(x(t'),t')\rangle$ in the corresponding Fourier space $(\mathbf{k},\omega)$ (Refs. 8 and 15)

$$\langle A_\alpha A_\beta\rangle_{k,\omega}=\frac{\coth(\hbar\omega/2k_BT)}{\sigma\omega}\left[\frac{k_\alpha k_\beta}{k^2}\right], \tag{3}$$

where σ is the conductance. This is valid for conducting systems at all temperatures where the fluctuations are strongly suppressed by the skin effect.[6] Following an approach similar to Chakravarty and Schmid,[7] the dephasing time τ_ϕ can be expressed using Eq. (3) by summing over all allowed frequencies and wave vectors:

$$\frac{1}{\tau_\phi}=\frac{e^2}{\sigma\hbar^2}\int du\int\frac{d\mathbf{k}}{(2\pi)^2}\int\frac{d\omega}{2\pi}\hbar\omega\,\coth\left(\frac{\hbar\omega}{2k_BT}\right)\times k^{-2}\exp(-Dk^2|u|-i\omega u). \tag{4}$$

In this expression, $1/\sigma$ for a quasi-1D wire is the resistance of a phase coherent length, $R_\phi=RL_\phi/L$. In a quasi-1D system, for any two interfering paths the wave vector $\mathbf{k}$ must be a two-dimensional vector.

Previous attempts to incorporate zero-point fluctuations only considered the motion of impurity ions.[16] Our approach differs from all others in that to calculate the low temperature saturation value of phase coherence time τ_0 we consider only the zero-point fluctuations of the intrinsic electromagnetic field. At low temperatures or high frequencies, $\omega\gg k_BT/\hbar$, the contribution to dephasing is dominated by the quantum part of the field fluctuations which diverges linearly for large ω. As in previous theories,[6–8] one must introduce a phenomenological upper cutoff to obtain any nondivergent fluctuation-dependent physical quantity. In the case of electrons in a quasi-1D disordered system, the fluctuations must be able to ''couple to the particle displacement'' (Refs. 9, 15, and 17) in order to contribute effectively to decoherence. The maximum frequency of the fluctuations that one should retain is no longer $k_BT/\hbar$, but becomes the average classical energy of the particle. For a ballistic sample this cutoff would then be the Fermi energy E_F, but for a diffusive sample it becomes $m^*v_D^2/2$, where m^* is the effective mass, v_D is the drift velocity of the particle given by v_Fl_e/L. l_e is the elastic mean free path, and, for a phase-coherent volume, L is essentially the phase-coherent length L_ϕ. Such a choice

of the upper cutoff is also natural in other models of decoherence, where the inverse of the collision timescale is typically taken as the cutoff.[17] Thus frequencies much higher than k_BT contribute to dephasing. In the problem at hand, $|\omega|$ in Eq. (4) is also bounded at the bottom by hD/L_ϕ^2. The electron traverses an average distance of L_ϕ before losing phase coherence. Therefore the maximum wavelength which can couple to the particle is $k^{-1}\sim L_\phi$. Wavelengths longer than L_ϕ do not contribute to dephasing. The lower cutoff in $|\omega|$ is given by $Dk^2\simeq D/L_\phi^2=1/\tau_\phi$.

Extending the ω integral over the range $h/\tau_\phi\leqslant|\hbar\omega|\leqslant m^*v_D^2/2$, and taking the low-temperature limit of the spectral density function $S(\omega)\rightarrow\hbar\omega/2$, the zero-point energy for the mode ω, from Eq. (4) one obtains

$$\frac{1}{\tau_0}=\frac{e^2R_\phi}{2\pi\hbar^2}\int_{hD/L_\phi^2}^{m^*v_D^2/2}d(\hbar\omega). \tag{5}$$

Except in very diffusive systems, the lower cutoff is small compared to the upper cutoff. Using $L_\phi(T\rightarrow0)=\sqrt{D\tau_0}$, the 1D zero-point phase coherence time τ_0 is given by

$$\frac{1}{\tau_0}=\left(\frac{e^2d^2Rm^*D^{3/2}}{4\pi\hbar^2L}\right)^2. \tag{6}$$

We have used $R/L=R_\phi/L_\phi$ and $D=v_Fl_e/d$, where D is the classical diffusion constant in d dimensions. Using the sample parameters given earlier, for the 1D wire displayed in Fig. 1, we find $\tau_0\simeq4.1$ nS, which is in good agreement with the measured saturation value of 3.6 nS. As shown in Ref. 4, this expression also accurately predicts the value of the low-temperature saturation phase coherence time τ_0 found in most published experiments on diffusive 1D wires including semiconductors. For a high mobility two-dimensional electron-gas wire with system size $L\simeq L_\phi$, from Eq. (4) one easily obtains an expression for the zero-point dephasing time $1/\tau_0=(m^*D/4h)(\Delta/\hbar)$ by using the Einstein relation $\sigma=e^2N(0)D$, where $\Delta=2/N(0)A$. $N(0)$ is the density of states at E_F, and A is the phase coherent area of the sample.

In the high temperature regime, Eq. (2) reduces to $S(\omega)=k_BT$.[6–9] The upper cutoff for ω can then be taken as k_BT, since the thermal fluctuations dominate with a thermal correlation time $\hbar/k_BT<\tau_0$. The crossover between the two regimes is very important to understand. Because of the finite frequency cutoffs in Eq. (5), in a phenomenological model one can approximate the integrand in Eq. (4) by the spectral density function for a single average energy mode, $\hbar\langle\omega\rangle\equiv\langle E\rangle$. The expression for the temperature dependence of τ_ϕ with a zero-point saturation time τ_0 becomes

$$1/\tau_\phi=(1/\tau_0)\coth(\langle E\rangle/2k_BT). \tag{7}$$

The above equation can be obtained from the rigorous solution of a Pauli Master equation for the decohering two-level system coupled to a thermal bath[18] for a single mode $\langle E\rangle/\hbar$. We interpret $\langle E\rangle$ to be the Thouless energy for a particle that diffuses over a volume defined by two diffusion length scales, the zero-point diffusion length $L_0=\sqrt{D\tau_0}$ and the thermal diffusion length $L_T=\sqrt{\hbar D/k_BT}$. The corresponding Thouless energy is then given by $\langle E\rangle=\pi hD/L_0L_T$. Equation (4) can then be written as $\tau_\phi=\tau_0\tanh[\alpha\pi^2\sqrt{\hbar/\tau_0k_BT}]$, where α is a constant of order unity which takes into account that our approximation for $\langle E\rangle$ is only good to first order. This is the same as Eq. (1), which was experimentally deduced[4] from studies of the dephasing time in 1D wires. Interestingly, the temperature dependence of τ_ϕ given by Eq. (1) at high temperatures reduces to that given by electron-electron interactions with large energy transfers.[6] As shown by the solid line in Fig. 1, Eq. (1) correctly describes the temperature dependence of the dephasing time for this sample as well as most 1D mesoscopic wires published to date once the phonon contribution to dephasing at high temperature is included by $1/\tau_\phi\rightarrow1/\tau_\phi+1/\tau_{ep}$, where τ_{ep} is the scattering time due to phonons. For 2D metal films, it has been shown recently[4] that Eq. (1) also correctly describes the temperature dependence of τ_ϕ, with the constant α reduced by a factor of π. For 1D semiconductor wires with $L_T\gg w$, the width of the sample, Eq. (1) becomes $\tau_\phi=\tau_0\tanh[(\alpha\hbar/k_BTw)\sqrt{D/\tau_0}]$, with τ_0 given by Eq. (6), and quantitatively describes the complete behavior observed in many published experiments. Note that at high temperatures for this case, $\tau_\phi\propto1/k_BT$, and not $(1/k_BT)^{1/2}$, as is the case for most metal wires.

It is interesting to note that Eq. (6) for the intrinsic dephasing time for mesoscopic wires τ_0 could also have been derived from a combination of the Aharonov-Bohm effect and the fluctuation-dissipation theorem. One very general way to write the fluctuation-dissipation theorem[15] is to relate the mean-square fluctuations $\langle F^2\rangle$ of any generalized force of a system in thermodynamic equilibrium with a parameter, $R(\omega)$ which characterizes an irreversible process,

$$\langle F^2\rangle=\frac{2}{\pi}\int R(\omega)S(\omega)d\omega, \tag{8}$$

where $S(\omega)$ is given by Eq. (2). Voltage across a resistive element is an example of a generalized force, and Eq. (8) reduces to the standard Johnson noise formula once the high-temperature limit of $S(\omega)$ is taken and $R(\omega)$ is identified with the resistance of the sample. In mesoscopics, the phase of the electron wave function is changed by a time dependent potential $V(x(t),t)$ by an amount given by the electrostatic Aharonov-Bohm effect, $\delta\phi=(e/\hbar)\int Vdt$. Following Stern, Aharonov, and Imry,[8] we define dephasing such that $\langle(\delta\phi)^2\rangle\sim1$, which occurs over a time scale of τ_ϕ. The dephasing time can be computed from

$$1\simeq\langle(\delta\phi)^2\rangle=\frac{e^2}{\hbar^2}\int_0^{\tau_\phi}\int_0^{\tau_\phi}dtdt'\langle V(t)V(t')\rangle. \tag{9}$$

Most of the contribution to $\langle(\delta\phi)^2\rangle$ will occur when $\omega\tau_\phi<1$,[8] and the environment can essentially be considered to be stationary over the time τ_ϕ. Equation (9) can then be approximated by $\langle V^2\rangle=\hbar^2/e^2\tau_\phi^2$ becoming the left-hand side of Eq. (8). The next important step is to remember that in mesoscopics, the conductance G of a phase-coherent volume fluctuates as the interference of all paths occurs at every point in the phase-coherent volume by an amount given by universal conductance fluctuation theory,[19] $\Delta G=e^2/h=\Delta R_\phi/R_\phi^2$ on average. Therefore we believe the parameter which controls the irreversibility in Eq. (8) is not the sample resistance R but rather the fluctuation $\Delta R_\phi=e^2R_\phi^2/h$. Since we are interested in the intrinsic decoherence time, we take

the low-temperature limit $S(\omega)\rightarrow\hbar\omega/2, \tau_\phi\rightarrow\tau_0$, and assume that $R_\phi(\omega)$ is frequency independent over the same limits of integration used in Eq. (5). Just as in the derivation of Eq. (6), we use $R/L=R_\phi/L_\phi$ and we easily arrive at the same result as given in Eq. (6).

In conclusion, we have shown that the phase coherence time in mesoscopic systems does not go to infinity as predicted by most theories, but will always be limited by the zero-point fluctuations of the intrinsic electromagnetic environment. We have shown that the ubiquitous saturation of τ_ϕ found in all low-temperature experiments on 1D mesoscopic systems can be quantitatively understood by incorporating the zero-point fluctuations into previous electron-electron interaction theories. We have demonstrated how the complete temperature dependence of τ_ϕ can be calculated using the full spectral density function for three examples, long metal wires, long semiconductor wires, and completely phase-coherent short wires. This functional form appears to be consistent with data from most experiments. In addition, we predict the magnitude of the low-temperature saturation value of this decoherence time from the knowledge of only R/L and the classical diffusion constant, in agreement with most experiments.

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