Coherent Signal Amplification in a Nanomechanical Oscillator via Stochastic Resonance

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Abstract. For the past few decades, stochastic resonance has emerged as a ubiquitous phenomenon, emerging in such diverse systems as solid-state electrical circuits and neurophysiological systems to large-scale climate modeling. The effect is attractive because it allows for the coherent amplification of a weak signal in a nonlinear system by the addition of a measured amount of white noise. Recently, a nanomechanical oscillator has been presented as a possible realization of a mechanical memory element. One of the major obstacles to widespread use of such elements is their behavior at higher temperatures, which has been seen to induce a deterioration of switch fidelity. The use of stochastic resonance provides a powerful means of counteracting such effects, as well as providing a tantalizing glimpse of the use of such phenomena in signal processing, quantum information and quantum control.

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Since its inception almost 25 years ago, the phenomenon of stochastic resonance[1,2] (SR) has proven itself to be remarkably powerful and universal. The basic requirements for a stochastically resonant system are an energy threshold (e.g. between two bistable states or ground and excited states), an external modulation which attempts to induce switching between the two states, and a source of noise. One central signature of the effect is a sharp peak in the signal-to-noise ratio (SNR) of the output signal from the system. The SNR is simply defined as the ratio of the spectral peak at the modulation frequency to the background signal at that same frequency. Despite these quite broad requirements, the observation of the phenomenon has been restricted to systems which are macroscopic in scale. With the development of a nanomechanical bistable system, however, it is now possible to observe the signature of stochastic resonance in a system with critical dimensions in the sub-micron range.

From the days of Euler, it has been known that a beam structure subjected to sufficient stress will begin to demonstrate nonlinear response. That is, it will evolve from behaving as a damped, driven, linear harmonic oscillator to a system more accurately described by the Duffing equation:

$$\ddot{x} + 2\gamma\dot{x} + kx + k_3x^3 = F\cos\omega t \qquad (1)$$

This response is hysteretic: sweeps up and down in frequency will show nonlinear jumps in the response at difference points. These jumps describe a range of frequencies in which the amplitude is multi-valued and the potential describing the beam response is double-well (as would be expected from a cubic restoring force in the equation of motion).

In previous work[3], we have shown that by driving a silicon doubly-clamped nanomechanical beam at low temperatures (300 mK) and through a magnetomotive technique, one can create a bistable nanomechanical oscillator. The occupation of the bistable states can then be controlled via the application of a modulation signal. Such an element possesses great potential to become the basis for the next generation of robust and fast memory elements, which will be mechanical in nature. Increasing the temperature has been seen[4] to have a deleterious effect on the switching coherence and fidelity.

The classic theory of stochastic resonance is centered on the idea that there is a matching condition between the timescale for transitions between the two states due to random fluctuations (the Kramers' rate) and the timescale imposed by the external modulation. The typical form for the Kramer's rate is given by:

$$\Gamma(D) \approx \exp(-\frac{\Delta V}{D}) \qquad (2)$$

Here, the noise strength D is from a Gaussian white noise source, and ΔV describes the potential barrier between the two states of the system. When the Kramers' rate is coupled with the periodic impulse

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provided by the modulation, there exists a condition whereby the period of the modulation matches this transition rate. When this occurs, the system will oscillate between its two states with a frequency equal to that of the modulation frequency. While this scenario is most dramatic when the modulation is too weak to induce transitions on its own, it has also been seen that there is an increase of the signal-to-noise ratio even when the modulation is sufficiently strong.

Figure 1 shows the results from our experiments on a bistable nanomechanical oscillator system. Figure 1(a) depicts the measurement circuit. The oscillator is excited at a frequency of 23.497 MHz, which is in the middle of the bistable region found through characterization with a vector network analyzer. It is then subjected to a square wave modulation of 0.05 Hz and a white noise signal obtained from an electronic synthesizer. The strength of the modulation is below the threshold necessary to achieve switching. The incident noise power is then increased in even increments while the response of the oscillator is monitored for a given length of time.

Figure 1(b) shows representative time scans of the beam response at three given noise powers. It is clear from the graphs that while the switching is sparse and erratic to begin with, both the number and coherence of the switches increase rapidly with increasing noise power. The results from Fast Fourier Transforms of each timescan occupy the right column of Figure 1(b). The increase of coherent response is clear from the power spectra. The SNR is calculated for each timescan in the normal way; these are plotted in Figure 1(c). While the SNR contains a plateau at its maximum and shows a very sharp drop-off, both of which are unusual, it is clear that it is the presence of increasing noise in this sample which allows for coherent signal amplification at modulation powers which are too weak to allow for any sort of switching.

The observation of stochastic resonance in this nanomechanical system is important for several reasons. Aside from simply adding one more system to the long list in which SR has been observed, these results open the door into the inclusion of stochastic resonance into the realm of nano-scale and quantum-mechanical structures. As device sizes continually shrink into this realm, the effects of noise and how to counteract them become even more important. Additionally, such devices are rapidly approaching the size scales dominated by quantum mechanics. Stochastic resonance may very well prove to be an important tool in achieving real control over quantum mechanical systems and devices.

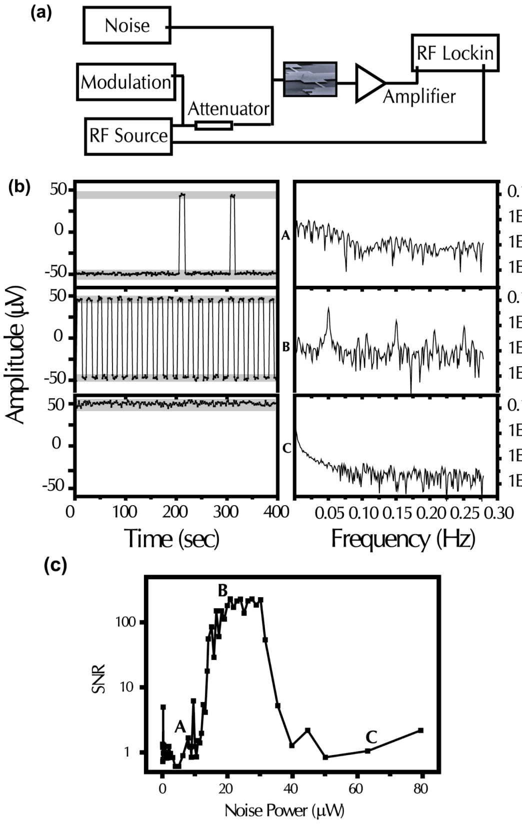

FIGURE 1. Experimental observation of stochastic resonance in a bistable nanomechanical oscillator. **(a)** Electrical circuit for excitation and modulation of the bistable state. **(b)** Response of the beam at the driving frequency, at three separate noise powers. The first panels (A) show sporadic switching. Increasing the noise (panels B) shows full and coherent switching and strong peaks in the power spectrum at the modulation frequency. Finally, increasing the noise still further (panels C) results in a loss of switching fidelity. **(c)** A plot of the experimental SNR. The letters on the curve match the panel sets in Fig 1(b).

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